Prototype of a small low noise absolute displacement sensor

Christophe Collette, Lionel Fueyo-Roza, Mihaita Horodinca

***Abstract*—Inertial seismic sensors have typically a velocity readout, in order to offer a large dynamic range in a large frequency range. However, in some precision engineering applications, it can be preferable to access to the displacement. An example is the so-called *sky-hook spring* strategy used for active vibration isolation. In this paper, we present a prototype of small inertial sensor with a displacement readout. It is based on a commercial low cost geophone, which has been modified to measure the displacement with a capacitive sensor. It results in a compact sensor with a resolution which is a factor 10 better than the commercial geophone, but a limited range of amplitudes. The paper finished with an attempt to extend the bandwidth of the sensor at low frequency.**

***Index Terms*—Absolute displacement sensor, geophone, capacitive sensor, inertial reference.**

I. INTRODUCTION

THE last fifty years have witnessed tremendous developments in seismometery [1]. Undoubtedly, the cornerstone of this evolution has been the introduction of the so-called *force balance principle* [2]–[5], which reduces the relative motion between the inertial mass and the support, and provides to feedback seismometers a much larger dynamic range than passive sensors. Actually, the introduction of this balancing force offers many advantages: (1) to increase the linearity of the sensor (because non-linear effects appear for large displacements), (2) to offer a flat sensitivity to acceleration or velocity at low frequency, which is better for measure seismic signals, and finally (3) to use a high resolution capacitive sensor to measure this relative motion. Since several decades, this principle is a standard practice in seismometers. Apart from seismometry, seismometers are also commonly used in Active Vibration Isolation (AVI) systems [6]–[8]. However, for this application, the amplitudes of interest are very small (between 1pm and 100nm), and the quantity of interest can be rather the absolute displacement. To some extend, seismometers, seismic accelerometers and geophones can be used for AVI systems, but further improvements require new developments: reduce the size, decrease the instrumental noise in the frequency range between $0.1\ Hz$ and $50\ Hz$, increase the robustness to environmental disturbances (e.g. stray magnetic field, radiation, temperature variations...), and at an affordable price.

Model	GS-11D	Nunmb. turns/coil	3680
Manufacturer	Geospace	Wire diameter	0.06mm
Sensitivity	32 V/(m/s)	Max current I max	90mA
Total weight	0.11 kg	Coils out. diam.	27.9mm
Inertial mass	0.018 kg	Stiffness	24 (N/m)
Coils int. diam.	25mm	Transducer const./coil	25N/A
Corner frequency	4.5 Hz	Damping ratio	.50
Resistance	2 kΩ		

TABLE I
CHARACTERISTICS OF THE GEOPHONE.

In this paper, we present a prototype of small inertial sensor with a displacement readout, to be used in AVI systems. It is based on a commercial low cost geophone, which has been modified in order to measure the displacement with a capacitive sensor. The geophone under test is a GS-11D, Geospace Technologies [9]. Its main properties are summarized in Table I.

The geophone principle is first reviewed in section two. Section three presents the prototype and the experimental results. Section four presents an active mean to extend the bandwidth of the sensor at low frequency. Section five draws the conclusions.

II. GEOPHONE PRINCIPLE

The working principle of the geophone is shown in Fig. 1(a). A coil is encircled around a seismic mass m, and connected to a resistance R. The ground w generates a relative motion between m and the coil. The relative motion creates a current i, and a voltage V_0 across the resistance.

The equations of the system are:

$$m\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) + Ti = 0 \tag{1}$$

for the mechanical part and

$$L\frac{di}{dt} - T(\dot{x} - \dot{w}) + Ri = 0 \tag{2}$$

for the electrical part, where i is the current, L is the inductance of the coil, and T is the constant of the coil, expressed in (Tm) or $V/(m/s)$.

Defining $y = x - w$, we get in the Laplace domain

$$ms^2Y + csY + kY + TI = -ms^2W \tag{3}$$

$$LsI - TsY + RI = 0 \tag{4}$$

The output of the sensor is the voltage V_0 across the resistance R, $V_o = RI$. Assuming that $R \gg sL$, the sensitivity of the geophone is given by

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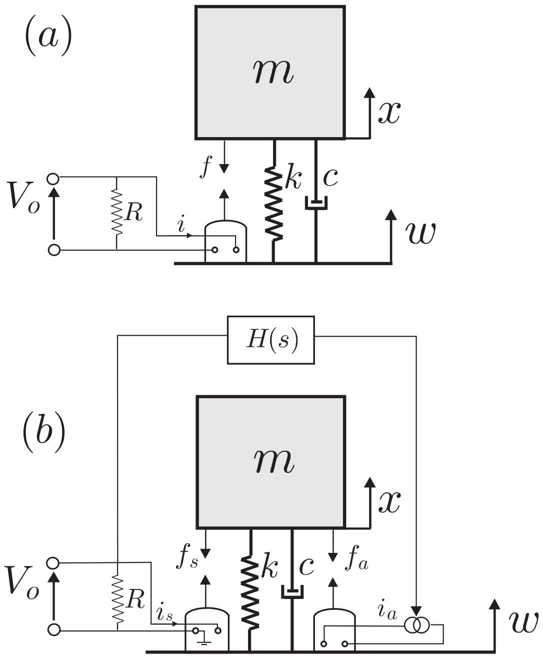

Fig. 1. Working principle of (a) a passive geophone and (b) a feedback geophone.

$$\frac{V_o}{sW} = \frac{-mTs^2}{ms^2 + s(c + \frac{T^2}{R}) + k} \tag{5}$$

or equivalently,

$$\frac{V_o}{sW} = \frac{-Ts^2}{s^2 + 2\xi_0\omega_0 s + \omega_0^2} \tag{6}$$

which is the typical expression of a high pass filter, where $\omega_0 = \sqrt{k/m}$ and $\xi_0 = (c + \frac{T^2}{R})/(2m\omega_0)$. In a real sensor, V_0 is polluted by several sources of noise, which are essentially characteristics of the mechanical and electrical components of the sensor (Brownian motion of the seismic mass, Johnson noise, current noise,..) [11], [12]. In this paper, all of the noise contributions are lumped in a quantity N. In this case, Equ.(6) becomes

$$V_0 = S(s)W(s) + N(s) \tag{7}$$

where

$$S(s) = \frac{-mTs^3}{ms^2 + s(c + \frac{T^2}{R}) + k} \tag{8}$$

is the sensitivity of the geophone expressed in (V/m). Equ.(7) shows that the smallest detectable quantity is limited by the sensor noise N. In practice, N can be measured by combining the output signals of two identical sensors placed side by side [13], [14]. Figure 2 shows the sensitivity curve of the geophone GS-11D, in units of (V/m). Typically, geophones can measure the velocity from a few Hertz to one hundred Hertz. At high frequency, the performances are limited by the higher order modes [10]. At low frequency, the performances are limited by the fundamental resonance of the inertial mass.

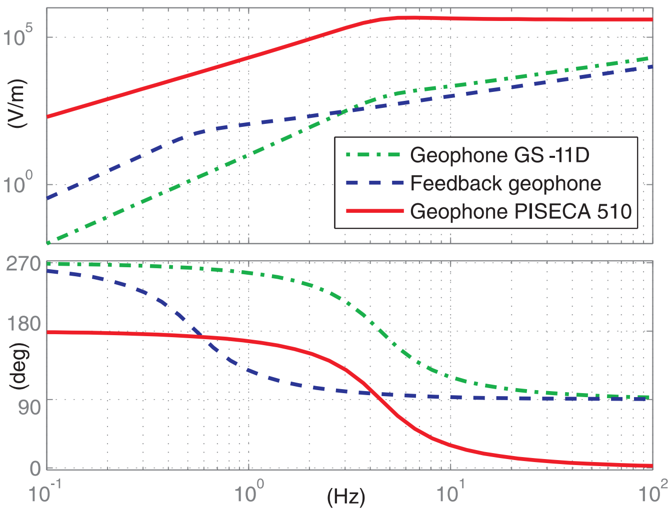

Fig. 2. Normalized sensitivity expressed in (V/m) of three sensors: a geophone GS-11D, an absolute displacement sensor (geophone PISECA 510) and a feedback geophone.

Figure 3 shows the power spectral density of the vertical displacement measured on a table in the Active Structures Laboratory at the University of Brussels with the geophone GS-11D, along with the instrumental noise of the GS-11D calculated with two geophones side by side.

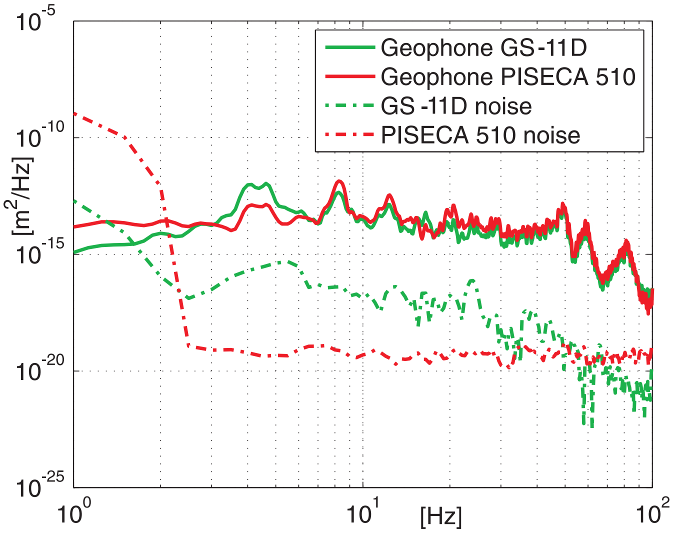

Fig. 3. Power spectral density of the vertical displacement measured on a table in the Active Structures Laboratory at the University of Brussels with a geophone GS-11D, with a geophone PISECA 510, instrumental noise of the geophone GS-11D, noise of the capacitive sensor.

One sees from Fig.3 that the noise of the geophone GS-11D is about 10^{-16} m^2/Hz, which is far from the target values presented in the introduction. In the next section, we present a modification of the GS-11D, where a capacitive sensor is mounted to measure the ground motion.

III. CAPACITIVE GEOPHONE

In seismometers, double capacitive sensors are commonly mounted symmetrically around the seismic mass in order to increase the linearity of the measurement, which is further increased when the measure is used in a feedback loop to restrict the relative motion between the seismic mass and the support. Such arrangement is vital for sub-Hertz applications, where a large dynamic range is required.

A direct measurement of the relative displacement between the casing and the inertial mass may be also useful for several other purposes. For example, it can be used to study the effect of a geophone inclination with respect to the gravitational field. A possible embodiment is disclosed in [15]. In [16], [17] a home made capacitive sensor has been mounted in a GS-11D to develop an affordable feedback accelerometer. In [18], two capacitive sensors are used in a piezoelectric low frequency feedback displacement sensor.

In this study, we measure the displacement of the inertial mass to develop a compact, absolute displacement sensor with a nanometer resolution. The cover of a GS-11D has been removed, and a thin blade has been attached on the middle part of the cylinder which supports the coil. The geophone and a capacitive sensor PISECA 510 from [19] have been mounted on a new dedicated support, shown in Fig. 4(a). The capacitive sensor, further connected to a signal conditioner PISECA E-852 from the same company, measures the relative displacement between the support and the blade, i.e. the inertial mass. For simplicity, it will be called the geophone PISECA 510 in the remaining of this paper. The theoretical sensitivity is shown in Fig. 2. The noise curve of the capacitive sensor is shown for comparison in Fig.3. It has been obtained by recording the signal while the capacitive sensor was pointing a fixed surface. One sees that the resolution of the geophone PISECA 510 is about 10^{-19} m^2/Hz, i.e. ten times as low as the resolution of the geophone GS-11D.

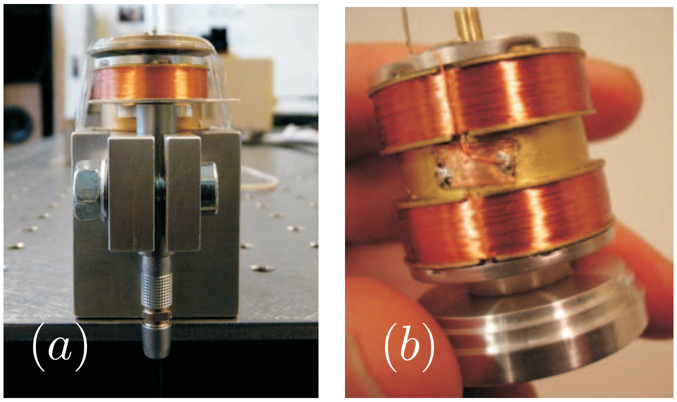

Fig. 4. (a) Geophone PISECA 510; (b) Feedback geophone.

The figure also shows the power spectral density of the vertical displacement measured on a table in the Active Structures Laboratory at the University of Brussels, and recorded at the same time as the geophone GS-11D. One sees that the power spectral densities of the two signals are very similar, even though a small mismatch is visible at very low frequency. The equivalence between the two sensors is confirmed by the excellent coherence between the two signals, shown in Fig.5.

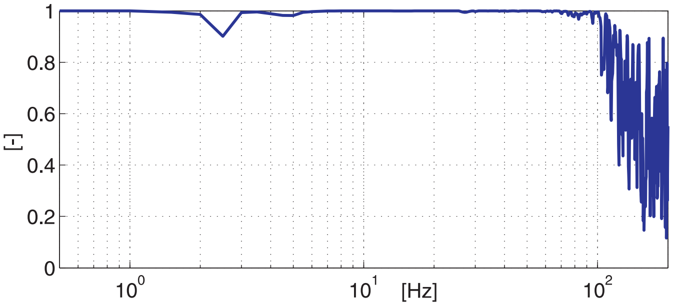

Fig. 5. Coherence between the signals recorded by the geophone GS-11D and the signals recorded by the geophone PISECA 510.

In the next section, we investigate a method to further improve the resolution at low frequency, by actively decreasing the corner frequency of the geophone in order to increase the sensitivity of the inertial sensor at low frequency.

IV. BANDWIDTH EXTENSION

A convenient method to improve the apparent sensitivity of the geophone at low frequency is to use a stretcher filter, with a double pole at ω_c and a double zero at ω_0, where $\omega_c < \omega_0$ [20], [21]. However, if the stretcher increases the bandwidth where the sensitivity is flat, it does not improve the ratio signal/noise of the geophone in the extended frequency range. Instead of the stretcher, consider a feedback geophone as shown in Fig. 1(b), where the coil is divided in two parts. One part is still used as sensor, and the other part is used as an actuator.In this case, Equ.(3) becomes

$$ms^2Y + csY + kY + T_aI_a + T_sI_s = -ms^2W \tag{9}$$

where T_s and T_a are the constants of the two parts of the coil. Using the same assumption that R is large, the output voltage is given by

$$V_o = RI_s \simeq T_s sY \tag{10}$$

where I_s is the current generated by the relative motion between the mass and the ground. Then, V_0 is used to generate a current in the actuator. Taking a classical Proportional plus Integral plus Derivative (PID) controller, we get

$$I_a = H(s)V_0 = (g_p + \frac{g_i}{s} + sg_d)V_0 \tag{11}$$

where g_p, g_i and g_d are the gains of the controller. Replacing (10) and (11) in (9) gives the new sensitivity

$$\frac{V_o}{sW} = \frac{-mT_s s^2}{(m + T_aT_sg_d)s^2 + (c + \frac{T_s^2}{R} + T_aT_sg_p)s + k + T_aT_sg_i} \tag{12}$$

The corner frequency of the geophone can be actively changed from $\sqrt{\frac{k}{m}}$ to $\sqrt{\frac{k+T_aT_sg_i}{m+T_aT_sg_d}}$ by choosing the values g_i and g_d. The proportional gain g_p is chosen to adjust the damping. In the useful bandwidth, the sensitivity becomes

$$\frac{V_o}{sW} = \frac{-mT_s}{m + T_aT_sg_d} \tag{13}$$

and the transfer function between the ground displacement and the relative displacement of the seismic mass is

$$\frac{y}{W} = \frac{-m}{m + T_a T_s g_d} \tag{14}$$

From (13) and (14), one sees that an additional feature of the relative acceleration feedback is that it can modify the sensitivity of the geophone. Choosing a positive value for g_d will force the seismic mass to move with the ground, and reduce the relative displacement of the seismic mass. As a consequence, the sensor will be able to measure much higher levels of vibrations without saturation, which is particularly useful to record strong earthquakes [22]. This is known as the *force balance principle*. On the other hand, choosing a negative value of g_d will increase the sensitivity of the sensor. In our case, we are only interested to increase the sensitivity at low frequency. Figure 6 shows the Nyquist plot of the open loop transfer function of the feedback geophone, obtained with the following numerical values: $g_d = 0$; $g_i = 0.75k$; $g_p = 0.6(c + T_s^2/R)$.

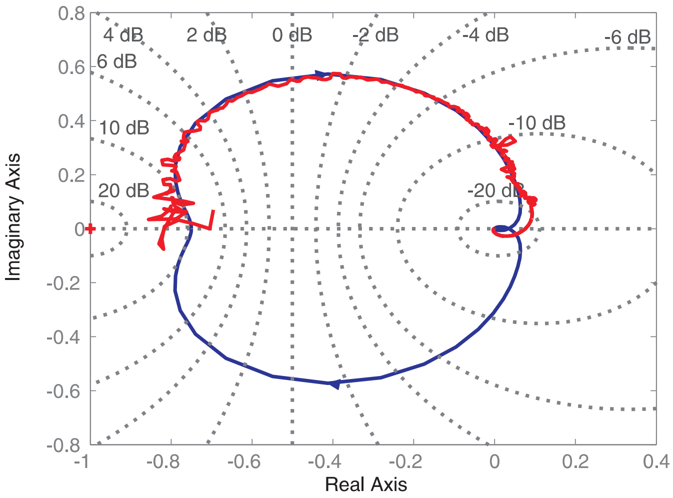

Fig. 6. Theoretical and experimental Nyquist plot of the open loop transfer function of the feedback geophone, obtained with 75% of negative stiffness.

The effect of the feedback operation on the sensitivity is shown in Fig. 7. The closed loop sensitivity is also shown in Fig. 2 in units of (V/m) for comparison.

This concept has been tested experimentally. The exterior cover of a second GS-11D was removed, which decreased the transduction constant, and the coil was separated in two independent parts (Fig. 4(b)): the sensor is connected to a digital control system, and the output is connected to the actuator through a current source. The experimental sensitivity curves in open loop and closed loop configuration are compared with the theoretical predictions in Fig. 7.

V. CONCLUSIONS AND PERSPECTIVES

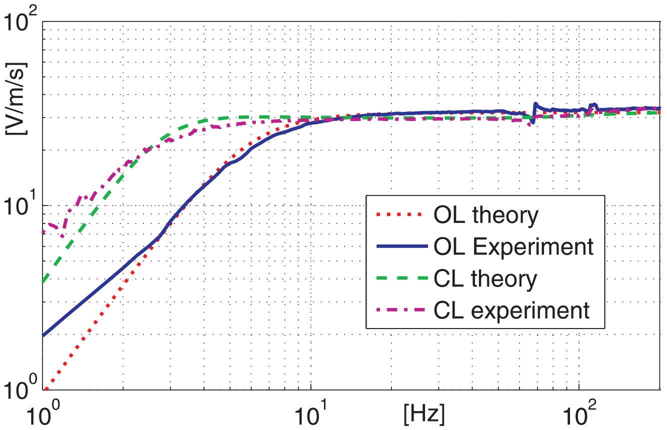

Fig. 7. Comparison of theoretical and experimental open loop (OL) and closed loop (CL) geophone sensitivity.

The objective of this study is to develop a new inertial sensor, compact, low cost, and with a sub-nanometer resolution in a frequency range between 1 Hz and 100 Hz. This paper presents a prototype of inertial sensor, being a first step towards this objective. The principle of the geophone has been first reviewed, before presenting the prototype. It consists of a geophone GS-11D in which a capacitive sensor has been mounted to measure directly the relative displacement between the support and the seismic mass. It results in an absolute displacement sensor above the fundamental resonance of the seismic mass on the membrane stiffness. The experimental results show that the readout of the new sensor correlates well with the readout of the non-modified geophone, and also that the resolution of the new sensor is improved by more than a factor 10 compared to the non-modified geophone.

Finally, a method has been presented in order to increase actively the sensitivity of the geophone at low frequency. The method has been test experimentally, confirming the theoretical predictions. Nevertheless, additional deep experimentations are still required before making the final decision for any future application of the method.

In a future work, the support of the capacitive sensor will be improved to allow a better alignment. An optical relative sensor is also foreseen as an alternative to the capacitive sensor, which has a limited range, and represents an expensive solution for the poor mechanics of the geophone, especially in regards of its thermal stability, which affects the permanent deflection of the seismic mass.

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