Online Control-Based Continuation of Nonlinear Structures Using Adaptive Filtering

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Summary. Control-Based Continuation uses feedback control to follow stable and unstable branches of periodic orbits of a nonlinear system without the need for advanced post-processing of experimental data. CBC relies on an iterative scheme to modify the harmonic content of the control target and obtain a non-invasive control signal. This scheme currently requires to wait for the experiment to settle down to steady-state and hence runs offline (i.e. at a much lower frequency than the feedback controller). This paper proposes to replace this conventional iterative scheme by adaptive filters. Adaptive filters can directly synthesize the control target adequately and can operate online (i.e. at the same frequency as the feedback controller). This novel approach is found to significantly accelerate convergence to non-invasive steady-state responses to the extent that the structure response can be characterized in a continuous amplitude sweep. Importantly, the stabilizing effect of the controller is not affected.

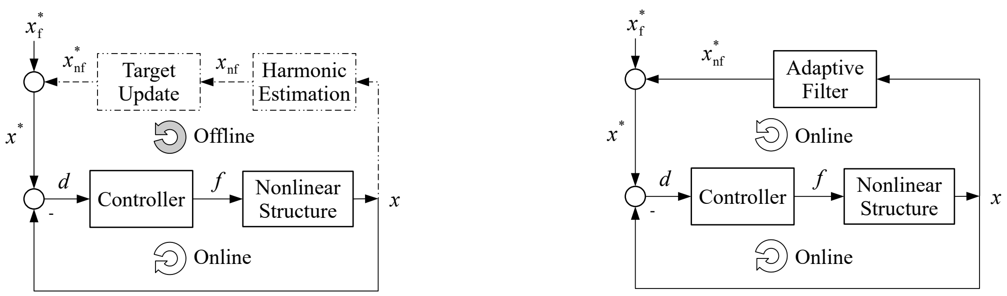
State of the art of Control-Based Continuation

A common way to characterize the steady-state behavior of a nonlinear structure consists in identifying its orbits under a monoharmonic excitation. Nonlinear structures can reach different periodic orbits under identical excitation, each one with its own response amplitude, stability and harmonic content. Some of the responses are unstable and cannot be observed experimentally without diverging towards another periodic orbit. Control-Based Continuation (CBC) is an experimental method that stabilizes the structure using feedback control of the displacement to generate the excitation signal and reach these unstable response branches [1][2]. Characterizing the unstable branches is useful to uncover potential hidden branches that, even though stable, cannot be reached by performing standard uncontrolled experiments such as frequency sweeps [3].

Non-fundamental harmonic content of the displacement x is fed back through the controller and must be taken into account to ensure monoharmonic excitation, synonymous with non-invasiveness of the controller [4]. Current implementations of CBC shown in Fig. 1a iterate (e.g. using Newton or fixed-point iterations) on the harmonics of the non-fundamental component of the control target x^*_{nf} until the non-fundamental harmonics of the excitation signal are below tolerance [5][6]. The fundamental component of the target x^*_{f} determines the fundamental amplitude of the response. Such a method necessitates waiting for steady-state to perform the harmonic decomposition before each target update.

Control-Based Continuation with adaptive filtering

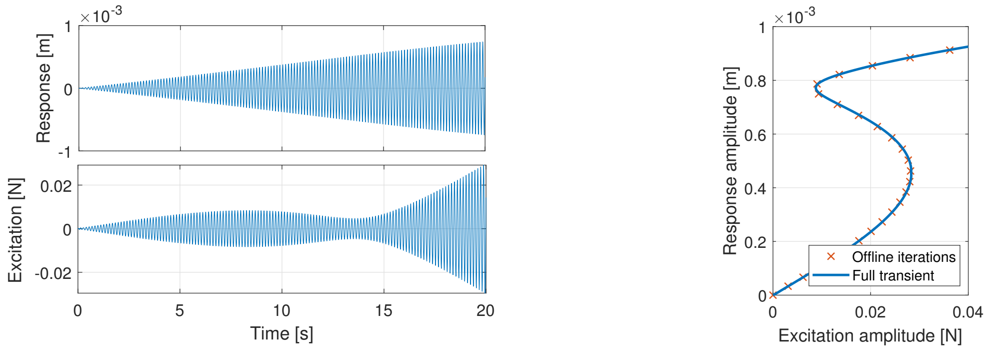
Adaptive filters have been used for online harmonic elimination in the literature [7]. In this work, they are used to estimate the non-fundamental harmonics of the response and synthesize the non-fundamental control target x^*_{nf}. The now fully online strategy is shown in Fig. 1b. No offline iteration is needed to ensure a non-invasive controller. Two continuation procedures can be considered: finite steps like the state of the art lead up to 25% of experimental time reduction; continuous sweep of the target amplitude lead up to 50% time reduction, but high sweep rate can induce transient effects in the system's dynamics, decreasing accuracy. When using an adaptive filter in closed-loop, the structural and filter dynamics are coupled. More research is needed to predict the effect of such a coupling. However, the convergence time of the adaptive filter in open-loop is significantly shorter than the structural transient time.



(a) Offline iterations on the non-fundamental target x^*_{nf}; the dashed-dotted line is performed offline, i.e. at a much lower frequency (state of the art)

(b) Fully online strategy to synthesize the non-fundamental target x^*_{nf}

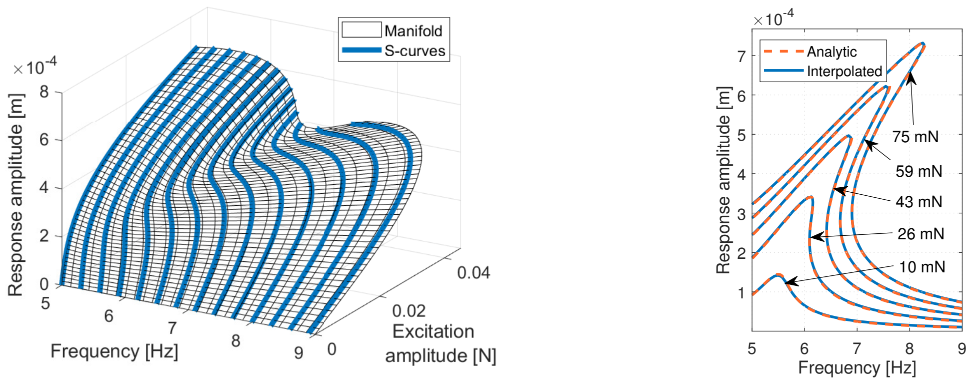
Figure 1: Two methods canceling high harmonic content in the excitation signal f by ensuring that the non-fundamental target x^*_{nf} equals the non-fundamental part of the response x_{nf}, making d monoharmonic; the fundamental target x^*_{f} drives the amplitude of response



(a) Time series of the target sweep at 7 Hz

(b) S-curve at 7 Hz

Figure 2: The relationship between excitation and response amplitudes (S-curve) can be obtained by sweeping the target amplitude



(a) S-curves and interpolating manifold in 3D space

(b) Frequency response curves

Figure 3: S-curves are measured for different frequencies and the manifold is interpolated using a B-spline surface; the interpolated frequency response curves are obtained by slicing the surface in the frequency-response plane for constant excitation amplitudes

The proposed strategy is demonstrated numerically using a Duffing oscillator; the resulting time series and the corresponding S-curves at 7 Hz are shown in Fig. 2. By performing sweeps at different frequencies, the manifold characterizing the structural dynamics can be interpolated, as shown in Fig. 3a. The frequency response curves can then be extracted, they are shown in Fig. 3b.

Conclusion

An online method that identifies stable and unstable periodic orbits of nonlinear mechanical systems is proposed. To ensure non-invasiveness of the controller, an adaptive filter is used to synthesize a target which renders the controller's output monoharmonic. This strategy removes the need for offline iterations on the control target. Therefore, identifying complete frequency response curves with fully-transient sweeps is within reach, significantly accelerating the experimental characterization of nonlinear systems.

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